

V_{US} FROM $K_{\ell 3}$ DECAYS

FEDERICO MESCIA

*Dip. di Fisica, Univ. degli Studi Roma Tre, Via della Vasca Navale 84, I-00146 Roma, Italy
INFN, Laboratori Nazionali di Frascati, Via E. Fermi 40, I-00044 Frascati, Italy*

Important progress made this year, both in theory and in experiment, helped solving the problem of 2σ -deviation from the unitarity of the first row elements in the CKM matrix. Today we have, $|V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 - 1 = -0.0008(13)$, $-0.0010(13)$, or $-0.0005(13)$, depending on whether the q^2 -dependence of the relevant $K_{\ell 3}$ form factor is considered as pole-like, linear or quadratic function, and on the Leutwyler-Roos value of $f_+(0) = 0.961(8)$, whose validity has recently been reinforced by lattice studies. In this talk we summarize the recent developments.

1 Introduction

$|V_{ud}|$ and $|V_{us}|$ are fundamental parameters of the Standard Model. The Cabibbo–Kobayashi–Maskawa (CKM) unitarity implies that $|V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 = 1$. In this equality, $|V_{ub}|$ is negligible in size, whereas $|V_{ud}|$ and $|V_{us}|$ induce comparable uncertainties.

The two most important determinations of $|V_{ud}|$ come from nuclear $0^+ \rightarrow 0^+$ transitions, and from the neutron beta decays. With respect to the value quoted in PDG ¹, a sign error of the radiative corrections to the neutron beta decays has been recently corrected ⁷. Consequently, the updated average value for $|V_{ud}|$ now reads:

$$|V_{ud}| = 0.9740 \pm 0.0005. \quad (1)$$

Using this value and by imposing the CKM unitarity, the Cabibbo angle ($|V_{us}|$) amounts to

$$|V_{us}|^{\text{uni.}} = 0.2265 \pm 0.0022. \quad (2)$$

Testing the unitarity of the 1^{st} -row of the CKM matrix means a comparison of this value with $|V_{us}|$ deduced directly from the processes governed by the $s \rightarrow u$ transition. Although theoretical constraints on $|V_{us}|$ from the semileptonic hyperon decays ^{2,3,4}, $\tau \rightarrow K\nu_\tau$ ⁵ and leptonic kaon decays ^{6,7} ($K_{\ell 2}$) recently became promising too, the best determination of $|V_{us}|$ is still obtained from $K \rightarrow \pi\ell\nu$ decay modes ($K_{\ell 3}$).

Before concentrating on the semileptonic $K_{\ell 3}$ decay, it is important to mention the intensive activity within the lattice QCD community invested in reducing the errors on the estimate of

f_K/f_π (cfr ref. ⁶). Once combined with the experimentally established $\Gamma(K \rightarrow \mu\nu)/\Gamma(\pi \rightarrow \mu\nu)$, this would allow for a precise determination of $|V_{us}/V_{ud}|$, and thus of $|V_{us}|$. This is why the experimenters recently became more interested in increasing the accuracy in measuring the $K_{\mu 2}$ decay rates ⁸. It should be stressed, however, that the current accuracy on $f_K/f_\pi - 1$ is about 6.5%, which amounts to a relative error of 1.2% for $|V_{us}|$. Therefore to achieve the challenging $\delta|V_{us}|/|V_{us}| = 0.1\%$, the relative error of $f_K/f_\pi - 1$ should be 0.5% or less, which is hardly feasible. In $K_{\ell 3}$ decay, instead, the equivalent requires a theoretical uncertainty of 7%, thanks to the conservation of the vector current (CVC) and the Ademollo–Gatto theorem (AGT) ⁹. Such an accuracy is within reach for the forthcoming lattice QCD studies.

2 $K_{\ell 3}$ decay modes and $|V_{us}|$

We first recall to the master formula for the $K_{\ell 3}$ decay rate:

$$\Gamma(K_{\ell 3}(\gamma)) = \frac{G_F^2 M_K^5}{128\pi^3} C_K^2 S_{\text{ew}} |V_{us}|^2 f_+(0)^2 \quad (3)$$

$$I_K^\ell(\lambda_{+,0}) \left(1 + \delta_{SU(2)}^K + \delta_{\text{em}}^{K\ell}\right)^2.$$

C_K^2 is equal to 1 (1/2) for the neutral (charged) kaon decay; $I_K^\ell(\lambda_{+,0})$ is the phase space integral defined in absence of electromagnetic corrections and depending on the slope parameters $\lambda_{+,0}$ which will be discussed below; $S_{\text{ew}} = 1.0232(3)$ is the universal short-distance electromagnetic correction ¹⁰ evaluated at $\mu = M_\rho$; $\delta_{\text{em}}^{K\ell}$ and $\delta_{SU(2)}^K$

	K_{e3}^+		K_{e3}^L			K_{e3}^S	$K_{\mu 3}^L$	
	BNL	NA48(*)	KTev	KLOE(*)	NA48	KLOE(*)	KTev	KLOE(*)
Br[%]	5.13(10)	5.14(6)	40.67(11)	39.85(35)	40.10(45)	0.0709(11)	27.01(9)	27.02(25)
Linear Parameterization								
I_K^ℓ	0.10627(15)		0.10337(15)			0.06877(16)		
$ V_{us} f_+(0)$	0.2185(23)	0.2187(15)	0.2155(9)	0.2133(13)	0.2140(15)	0.2160(16)	0.2148(10)	0.2148(14)
Pole Parameterization								
I_K^ℓ	0.10580(15)		0.10291(15)			0.06820(18)		
$ V_{us} f_+(0)$	0.2190(23)	0.2192(15)	0.2160(9)	0.2138(13)	0.2145(15)	0.2164(17)	0.2157(11)	0.2157(14)
Quadratic Parameterization								
I_K^ℓ	0.10520(71)		0.10233(70)			0.06777(48)		
$ V_{us} f_+(0)$	0.2196(24)	0.2198(17)	0.2166(12)	0.2144(15)	0.2151(17)	0.2171(17)	0.2164(13)	0.2164(16)

Table 1. Recent results from BNL-E865¹¹, KTeV¹², NA48¹⁴ and KLOE⁸ and corresponding values of $|V_{us}|f_+(0)$. Preliminary results are marked by (*). We use for the linear parametrization $\lambda_+ = 0.0281(4)$ and $\lambda_0 = 0.017(1)$, for the pole one $[f_{+,0}(t) = f_+(0)/(1 - \lambda_{+,0} t/m_{\pi^+}^2)]$, $\lambda_+ = 0.0250(4)(4)$ and $\lambda_0 = 0.014(1)$ from¹², and for the quadratic one $[f_0(t) = f_+(0)(1 + \lambda_0 t/m_{\pi^+}^2)]$, and $f_+(t) = f_+(0)(1 + \lambda'_+ t/m_{\pi^+}^2 + \lambda''_+ t^2/(2 m_{\pi^+}^4))$ $\lambda_0 = 0.0137(13)$, $\lambda'_+ = 0.0206(18)$, and $\lambda''_+ = 0.0032(7)$ from¹². In addition, $\tau_{K_L}^{PDG} = 5.15(4) \times 10^{-8} s$, $\tau_{K^+}^{PDG} = 1.2384(24) \times 10^{-8} s$ and $\tau_{K_S}^{PDG} = 8.953(8) \times 10^{-11} s$ are used along with $\delta_{em}^{K\ell}$ for the fully inclusive rate.

are respectively the long-distance electromagnetic and strong isospin-breaking corrections; finally, $f_+(0)$ is the vector form factor at zero momentum transfer [$t \equiv q^2 = (p_K - p_\pi)^2 = 0$] which encodes the SU(3) breaking effects in the hadronic matrix element. To extract the value of $|V_{us}|$ from eq. (4) one needs not only an accurate experimental values for the rate (Γ) and for the kinematic integral I_K^ℓ , but also the theoretical estimates of the δ 's and $f_+(0)$. In what follows, we provide the update to each of these quantities.

Width measurements: This summer, all the new generation kaon experiments released results for the $K_{\ell 3}$ decay modes. The important novelty is that these new results are consistent among themselves (see table 1), but they disagree with the old ones.

$I_K^\ell(\lambda_{+,0})$ and the form factor shapes: KTeV¹², ISTRA+¹³ and NA48¹⁴ studied the t -dependence of the partial $K_{\ell 3}$ rates: KTeV and ISTRA+ analyzed both muonic and electronic decays, while NA48 restrained to K_{e3}^L only. Dalitz plot data have been examined by assuming a linear, quadratic and pole dependence in t . With the linear function, $f_{+,0}(t) = f_+(0)(1 + \lambda_{+,0} t/m_{\pi^+}^2)$, the three groups agree on the values for the slopes, $\lambda_{+,0}$, which are more accurate than the ones reported by the PDG. The average is $\lambda_+ = 0.0281(4)$, and $\lambda_0 = 0.017(1)$. Concerning the

presence of the quadratic term in $f_+(t)$, findings are controversial: contrary to NA48, KTeV and ISTRA+ collaborations observe a non-zero curvature, but with 1σ significance only. A closer look at the systematics, and the results by KLOE are certainly needed. Finally, the pole fit, tried by both KTeV and NA48, looks, for the time being, the most reasonable solution and the measured pole mass is consistent with the mass of $K^*(892)$ (as anticipated by the lattice study, earlier this year²¹). In the case of the scalar form factor $f_0(t)$ no curvature has been observed.

Strong and em isospin breaking effects: $\delta_{em}^{K\ell}$

	$\delta_{SU(2)}^K(\%)$	$\delta_{em}^{K\ell}(\%)$	
		3-body	full
K_{e3}^+	2.31(22)	-0.35(16)	-0.10(16)
K_{e3}^0	0	+0.30(10)	+0.55(10)
$K_{\mu 3}^+$	2.31(22)	-0.05(20)[*]	+0.20(20)[*]
$K_{\mu 3}^0$	0	+0.55(20)	+0.80(20)

Table 2. Summary of the isospin-breaking factors^{15,16,17}: $\delta_{em}^{K\ell}$ [3 body] denotes corrections for the inclusive rate involving radiative events inside the $K_{\ell 3}$ Dalitz Plot, whereas $\delta_{em}^{K\ell}$ [full] those for the fully inclusive $K_{\ell 3(\gamma)}$ rate. The entries with [*] are from ref.¹⁷.

and $\delta_{SU(2)}^K$ corrections have been recently and properly calculated at $O[(m_d - m_u)p^2, e^2 p^2]$ in^{15,16}. The numerical results are collected in

table 2.

$|V_{us}|f_+(0)$ estimates: With the three ingredients discussed so far, we can extract $|V_{us}|f_+(0)$ with small theoretical errors from both charged and neutral modes (see table 1), allowing us a first consistency check¹⁵ between experiment and theory. In fig. 1 we show the points obtained by assuming the pole-like t -dependence for the form factors with the corresponding pole masses determined by KTeV. The resulting experimental

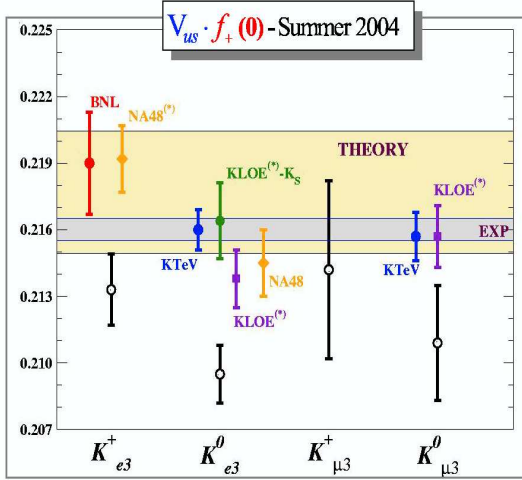


Figure 1. Results of $|V_{us}|f_+(0)$. Full points are obtained by assuming the pole-fit choice and recent measures (see table 1). Empty points are on the older experiments¹. The “EXP” and “THEORY” bands indicate respectively the average of the new experimental results (eq. (4)) and the unitarity prediction in eq. (9).

average reads

$$(|V_{us}|f_+(0))^{exp} = 0.2160 \pm 0.0005, \quad (4)$$

which is represented in fig. 1 by the dark-shaded band. Had we used the linear (quadratic) parameterization, the central value would shift by -0.07% ($+0.06\%$). At this conference it was argued⁸ that the small difference between the values of $|V_{us}|f_+(0)$ as extracted from $K^+_{\ell 3}$ and from $K^0_{\ell 3}$ might be due to a problem of the present value of the K_L -lifetime.

SU(3) breaking effects and $f_+(0)$: The remaining ingredient to extract $|V_{us}|$ from eq. (4) is

$f_+(0)$. This quantity is the origin of the largest uncertainty in $|V_{us}|$, namely $\delta|V_{us}|/|V_{us}| \simeq 1\%$, to be compared with 0.2% and 0.35% coming from the isospin breaking corrections and the uncertainty on the phase space integral, respectively. According to eq.(4), $f_+(0)$ is defined in the absence of em and strong isospin-breaking terms and incorporates only strong SU(3)-breaking effects. Its expansion in chiral perturbation theory (ChPT) reads,

$$f_+(0) = 1 + f_2 + f_4 + \dots, \quad (5)$$

where $f_+(0) = 1$ reflects the CVC in the SU(3) limit, while f_2 and f_4 stand for the leading and next-to-leading chiral corrections. AGT ensures that the SU(3) breaking corrections are quadratic in $(m_s - m_u)$ and $f_2 = -0.023$ is a clean prediction by ChPT, i.e. no unknown couplings enter at $O(p^4)$. The calculation of the chiral loop contribution, $\Delta(\mu)$ in

$$f_4 = \Delta(\mu) + f_4^{loc}(\mu), \quad (6)$$

has been recently completed^{18,19}. The estimate of f_4 , however, still suffers from the uncertainty due to the lack of knowledge of the low energy constants entering $f_4^{loc}(\mu)$. The PDG quotes the value obtained in the quark-model calculation by Leutwyler-Roos²⁰ (LR),

$$f_4 = -0.016 \pm 0.008 \rightarrow f_+(0) = 0.961(8), \quad (7)$$

based on parameterization of the asymmetry between kaon and pion wave functions. If the estimate of $f_+(0)$ in eq. (7) is used along with the experimental average of $|V_{us}|f_+(0)$, eq. (4), one gets

$$|V_{us}| = 0.2248 \pm 0.0018_{f_+(0)} \pm 0.0005_{exp}, \quad (8)$$

in good agreement with the value obtained by imposing the CKM unitarity [cfr eq. (2)]. This compatibility is also observed in fig. 1 where the light-shaded band refers to,

$$|V_{us}^{uni.}|f_+(0)_{eq.(7)}^{theo.} = 0.2177 \pm 0.0028. \quad (9)$$

The LR estimate has been corroborated this year by a (quenched) lattice QCD study that gave²¹

$$f_4 = -0.017 \pm 0.009 \rightarrow f_+(0) = 0.960(9). \quad (10)$$

In this estimate, the leading quenched artifacts have been subtracted, but residual effects at $\mathcal{O}(p^6)$ are still present. A conservative uncertainty of 60% has been attributed to f_4 ²¹, which can be substantially reduced by an unquenched calculation. Besides the lattice estimates, two more calculations^{18,22} appeared this year, yielding respectively:

$$f_4 = -0.001 \pm 0.010 \rightarrow f_+(0) = 0.976(10), (11)$$

$$f_4 = -0.003 \pm 0.011 \rightarrow f_+(0) = 0.974(11). (12)$$

However they both contain model-dependent assumptions. In particular a strong ansatz to get $f_4|^{loc}$ has been imposed in¹⁸, which then propagates to ref.²² where the value (11) is used as input. Specifically, the authors of¹⁸ identify the LR value of $f_4 = -0.016(8)$ with $f_4|^{loc}(\mu)$ at the scale $\mu = m_\rho$. For the estimate of $f_+(0)$ this means adding the loop contribution $\Delta(\mu = m_\rho) = 0.015$ to the LR value. Such an interpretation of the LR result is questionable: the choice $f_+(0) = 0.961(8) + \Delta(\mu)$ could be carried out at a different scale. In this case, by varying μ in a reasonable range [0.5 GeV-1 GeV]: $\Delta(\mu) = 3.5\% \rightarrow 0.4\%$ and $f_+(0) = 0.996(8) \rightarrow 0.965(8)$. Because of this scale uncertainty, the error bars in eqs. (11,12) should be considerably larger (see comment in¹⁵). Notice also that by using the values of $f_+(0)$ (11) and (12), unitarity is violated by about $+1.4\sigma$. The corresponding $|V_{us}|f_+(0)$ theory band in fig. 1 would be shifted to $|V_{us}^{uni}|f_+(0) = 0.221(3)$, i.e. consistent with the K^+ experimental values, but well above the K^0 ones.

Before concluding we should stress that in literature^{14,23} the value $f^{K^0\pi^-}(0) = 0.981 \pm 0.010$, and $f^{K^+\pi^0}(0) = 1.002 \pm 0.010$ of ref.¹⁵ are erroneously treated as independent estimates of $f_+(0)$, and directly compared to the ones discussed in this write-up. The apparent inconsistency is due to the fact that the above results refer to a different definition of $f_+(0)$, in which some of the isospin breaking corrections are included in the definition. Once we remove these corrections to perform a consistent comparison with the standard definition (used in this write-up, by the

PDG¹ and by KTeV¹²), the two above values give $f_+(0) = 0.976(10)$, which is the result quoted in eq. (11). Our analysis of $|V_{us}|$ gives exactly the same result of¹⁵ as long as the SU(3)-breaking estimates in the form factors are kept identical.

In conclusion, a novel route to estimate f_4 by means of lattice QCD has been devised this year. The quenched value essentially confirms the one obtained long ago by LR. This result and, more importantly, the new experimental data helped resolving the puzzle of the 1st row CKM-unitarity. To perform a more accurate test of CKM unitarity, an unquenched lattice QCD calculation of f_4 is needed. In a less near future, an alternative will be the proposal of ref.¹⁸, who showed that the couplings in f_4 can be determined from the precision measurement of the slope and curvature of the scalar form factor $f_0(t)$.

ACKNOWLEDGMENTS

We thank M. Antonelli, P. Franzini, G. Isidori and A. Sibidanov for discussions on the subject of this talk. I am also very grateful to all my friends and colleagues of the SPQCDR Collaboration. The work of F.M. is partially supported by IHP-RTN, EC contract No. HPRN-CT-2002-00311 (EURIDICE).

References

1. S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592** (2004) 1.
2. N. Cabibbo, E. C. Swallow and R. Winston, Phys. Rev. Lett. **92**, 251803 (2004) [hep-ph/0307214];
N. Cabibbo, E. C. Swallow and R. Winston, Ann. Rev. Nucl. Part. Sci. **53**, 39 (2003) [hep-ph/0307298].
3. R. Flores-Mendieta, hep-ph/0410171.
4. D. Guadagnoli *et al.*, hep-lat/0409048.
5. E. Gamiz *et al.*, hep-ph/0408044.
6. C. Aubin *et al.* [MILC Collaboration], hep-lat/0407028.

7. A. Czarnecki, W. J. Marciano and A. Sirlin, hep-ph/0406324.
8. P. Franzini, hep-ex/0408150;
M. Antonelli [KLOE Collaboration], at this conference.
9. M. Ademollo and R. Gatto, Phys. Rev. Lett. **13** (1964) 264.
10. A. Sirlin, Nucl. Phys. B **196**, 83 (1982).
11. A. Sher *et al.*, Phys. Rev. Lett. **91**, 261802 (2003) [hep-ex/0305042].
12. T. Alexopoulos *et al.* [KTeV Collaboration], hep-ex/0406003;
T. Alexopoulos *et al.* [KTeV Collaboration], hep-ex/0406001.
13. O. P. Yushchenko *et al.*, Phys. Lett. B **581**, 31 (2004) [hep-ex/0312004];
O. P. Yushchenko *et al.*, Phys. Lett. B **589**, 111 (2004) [hep-ex/0404030].
14. A. Lai *et al.* [NA48 Collaboration], hep-ex/0410059;
A. Lai *et al.* [NA48 Collaboration], hep-ex/0410065;
L. Litov [NA48 Collaboration], at this conference.
15. V. Cirigliano *et al.*, Eur. Phys. J. C **35** (2004) 53 [hep-ph/0401173];
V. Cirigliano *et al.*, Eur. Phys. J. C **23** (2002) 121 [hep-ph/0110153].
16. T. C. Andre, hep-ph/0406006.
17. E. S. Ginsberg, Phys. Rev. D **1** (1970) 229.
18. J. Bijnens and P. Talavera, Nucl. Phys. B **669**, 341 (2003) [hep-ph/0303103].
19. P. Post and K. Schilcher, Eur. Phys. J. C **25**, 427 (2002) [hep-ph/0112352].
20. H. Leutwyler and M. Roos, Z. Phys. C **25** (1984) 91.
21. D. Becirevic *et al.*, hep-ph/0403217.
22. M. Jamin, J. A. Oller and A. Pich, JHEP **0402**, 047 (2004) [hep-ph/0401080].
23. J. Charles *et al.* [CKMfitter Collaboration], hep-ph/0406184.